Are two-digit Arabic numbers processed according to the holistic model, when they are represented internally?

by Patricia Karsten

Abstract

The study explored how two-digit Arabic numbers are processed, when they are represented internally. The holistic model predicted unit and decade effects, but no discontinuity or reverse distance effects. A within-participants experimental design was employed with 50 participants of mixed age, gender and nationality. Participants determined the larger or smaller of two numbers. Response times showed a significant unit effect, but no decade effect. These results did not fully support prior research findings for internally represented numbers, which were obtained using fixed comparison standards. Further research may help to determine the conditions for a decade effect to appear.

Introduction

Research in numerical cognition is interested in the way numbers are processed, when humans perform mathematical operations, like the simple comparison task to decide which of two numbers is larger. Zhang and Wang (2005) distinguished holistic, sequential and parallel processing models.

According to the holistic model, two-digit numbers are not processed digit-wise, but as a whole, with reaction times smoothly decreasing with increasing unit and decade distance. By contrast, in sequential processing the decade digits are considered first, and the unit digits only if the decade digits do not suffice for a decision. Due to a general distance effect, reaction times decrease with increasing decade distance of two numbers, but they show discontinuities at decade boundaries. Finally, the parallel model assumes that decade and unit digits are considered simultaneously, giving rise to facilitatory and inhibitory effects on reaction times due to Stroop-like digit interference. These can overcompensate a distance effect to a certain extent, leading to reverse distance effects across decade boundaries (Zhang and Wang, ibid.).

Zhang and Wang (ibid.) found that reaction time patterns for external representations were inconsistent with the holistic model, as they showed reverse distance effects which suggested processing according to the parallel model. Reaction time patterns for mixed representations were more in line with the holistic model, but there were also signs for parallel processing. Their procedure for mixed representations was to first present the fixed comparison standard (55 or 65) on screen, and the target number after a short delay. It was assumed that the first number was internalized during the delay, and the second number then needed to be internalized as well, because the processing required for the comparison task was only possible for identical representations, resulting in internal representation of both numbers. Zhang and Wang (2005) attributed the unclear evidence with mixed representations to possibly incomplete internalization of the fixed comparison standard.

In their experiments with mixed representations, Dehaene, Dupoux and Mehler (1990) reported discontinuities at decade boundaries with fixed comparison standards 55 and 66, but not with 65.

Citation: Karsten, Patricia (2009) Are two-digit Arabic numbers processed according to the holistic model, when they are represented internally? retrieved from http://www.patriciakarsten.com

They concluded that the repeated digit attracted attention and thus influenced processing of numbers close to the standard.

This project focused on the question whether internally represented two-digit numbers are processed according to the holistic model, and addressed it in a slightly modified way. Also based on a comparison task, it used only internal representations and no fixed standard, and alternated the task (smaller or larger) at random. This approach aimed at increasing the ecological validity of the task and avoiding confounding effects from mixing representations and incomplete internalization of standards. Internal representation was supposed to be achieved by presenting two numbers first and the task afterwards, so that both numbers had to be memorized, and subsequent processing was based on an internal representation. It was predicted that processing would follow the holistic model, implying unit and decade effects, but no discontinuity or reverse distance effects. This led to the following research hypotheses:

Unit effect: Decision times for large unit distances differ from those for short unit distances

Decade effect: Decision times for large decade distances differ from those for small decade distances

Discontinuity effect: Decision times for number pairs that include a decade boundary differ from those close to a decade boundary. Decision times for number pairs with large decade distance differ from those with small decade distance. There is an interaction between closeness to decade boundary and decade distance.

Reverse distance effect: Decision times for number pairs with digit interference differ from those without digit interference. Decision times for number pairs with large decade distance differ from those with small decade distance. There is an interaction between digit interference and decade distance.

Method

Design

The experiment employed a within-participants design with one or two factors, depending on the effect. Unit and decade distances between two two-digit Arabic numbers were the independent variables, manipulated to meet the requirements for assessing the different effects. Participants' reaction time for a comparison decision, measured to the nearest ms, was the dependent variable.

For the unit effect, unit distance between two numbers represented the sole factor, levels 1 to 8. For the decade effect, sole factor was decade distance between two numbers, levels 1 to 8. The two factors for the discontinuity effect were 'closeness to decade boundary' with 3 levels (b-2, b-1, and b), and decade distance, levels 1 to 8. The two factors for the reverse distance effect were 'digit interference' with two levels (yes/no), and decade distance, levels 1 to 7.

Possible confounding effects from individual differences in mathematical decision speed and from fixed comparison standards were controlled through the within-participants design choice and by using flexible number pairs.

Participants

50 OU students and persons known to them personally, of mixed age, gender and nationality, were recruited as participants on a voluntary basis. They were supposed to be relatively naïve to the research question, have normal or corrected eyesight and normal short-term memory functioning.

Materials

The experiment was conducted using PCs and the E-Studio software. The stimuli consisted in a randomized sequence of 72 two-digit number pairs, 18 for each of the unit, decade, discontinuity and reverse distance effects.

Unit effect number pairs were on the same decade and differed on the unit distance, for example 11/12 or 11/19. Decade effect number pairs differed on the decade distance, for example 11/21 or 11/91. For the discontinuity effect, 6 sets of 3 related number pairs combined a base number with either the decade boundary, boundary -1 or boundary -2, for example 99/80, 99/79 and 99/78. For the reverse distance effect, 9 sets of 2 related number pairs were constructed, where one number pair had interfering digits and larger absolute distance, while the other had no interfering digits and smaller absolute distance, for example 26/45 and 33/45.

Procedure

Participants downloaded the experiment file and written instructions, which also contained briefing information and consent agreement.

E-Prime presented the trials in random order, clustered in 4 blocks of 18 number pairs, separated by pauses. The experiment was preceded by instruction screens and 12 practice trials, and followed by debriefing information after the last trial. In a single experimental trial, two numbers were presented on screen for 2 seconds, followed by the randomly chosen task (smaller or larger) after a delay of 500ms. The participant responded with pressing the z-key if the correct number was on the left side, and the m-key if it was on the right. The experiment took 6 to 8 minutes to complete, including pauses. E-Prime recorded participants' response data in a file, which was sent back as an electronic message to the experimenter.

Compliance with BPS ethical principles was sought by obtaining informed consent from participants through briefing information which explained the aims of the research and reassured participants about their right to withdraw at any time, and the confidentiality and anonymity of their data. A pilot study with 6 participants from the researcher's OU tutor group led to a substantial reduction in the number of trials and to the introduction of flexible-length pauses during the experiment, to assure participants' well-being.

Results

Incorrect responses and one data set with extreme values (9 times above average) were excluded, and 2 missing data values were replaced by their series means. Unit and decade distances were grouped into high (5-8) and low (1-4) to obtain clearer results. Means and standard deviations were calculated, and one- and two-way repeated-measures analyses of variance (ANOVA) performed, with an α-level of .05 for all tests.

A one-way ANOVA for the unit effect stimuli showed a significant effect. Mean response times were faster for high compared to low unit distances ($F_{1,48} = 7.462$, $p = .009$). There was also a significant downward sloping, linear trend ($F_{1,48} = 6.911$, $p = .011$), with decreasing mean response times from 821 to 695 ms, see figure 1.

A one-way ANOVA for the decade effect stimuli showed no significant effect. The 3x2 and 2x2 ANOVAs for the discontinuity and reverse distance stimuli showed no significant main effects, nor interactions. However, there was a close to significant decade effect in the reverse distance stimuli ($F_{1,48} = 3.442$, $p = .07$) with faster response times for high compared to low decade distances. Table 1 presents mean response times for all conditions.

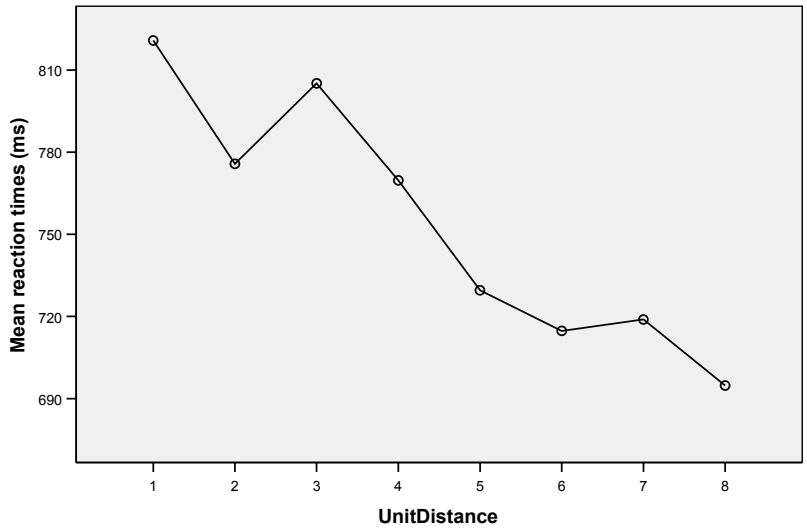

Figure 1: Mean reaction times (ms) by unit distances 1 to 8 in the unit effect stimuli

Table 1: Mean response times in ms for unit, decade, discontinuity and reverse distance effects

Unit effect	***High unit distance (5-8)***	***Low unit distance (1-4)***	***overall***
**overall*	711 (SD 149)	794 (SD 250)	753
Decade effect	***High decade distance (5-8)***	***Low decade distance (1-4)***	***overall***
overall	771 (SD 225)	749 (SD 156)	760
Discontinuity effect	***High decade distance (5-8)***	***Low decade distance (1-4)***	***overall***
boundary -2	801 (SD 382)	765 (SD 267)	783
boundary -1	749 (SD 269)	736 (SD 189)	743
boundary	706 (SD 238)	762 (SD 230)	734
overall	752	754	753
Reverse distance effect	***High decade distance (5-8)***	***Low decade distance (1-4)***	***overall***
interference	743 (SD 247)	788 (SD 243)	766
**no interference*	718 (SD 292)	782 (SD 221)	750
overall	731	785	758

*Note: n=49 participants in all conditions; SD = standard deviation; * indicates significant or close to significant effects.*

Discussion

The holistic model predicted unit and decade effects, but no discontinuity or Stroop-like reverse distance effects for comparisons of internally represented two-digit number pairs. The experimental results permitted to reject the null hypothesis for the unit effect, but not for the other effects. Still, the results seemed to grossly support the predictions of the holistic model: There was a significant unit effect, there were no significant discontinuity or Stroop-like reverse distance effects, and there were close to significant signs for a decade effect in the no-interference condition of the reverse distance stimuli.

On the other hand, it remained the question why there was no decade effect in the decade effect stimuli, and why signs of a decade effect appeared in the no-interference reverse distance stimuli, while absent in the interference condition. This latter result pointed to the possibility that reaction times were influenced by digit interference, hiding a decade effect in the interference condition. This would have contradicted the holistic model which posits that there is no interference from individual digits, and would have given support to the parallel model.

The decade effect stimuli had been constructed by distributing decade distances evenly over the range 1 to 8, and choosing two different random unit digits for each number pair. This did not exclude the appearance of interfering digits similar to those in the reverse distance stimuli. A retrospective analysis of the decade effect stimuli detected three different situations: Stimuli without interference, with partial interference and with full interference. A 3x2 ANOVA was then conducted, but showed no significant interference, decade effects or interaction, contradicting this attempt at explaining the lack of a decade effect.

Overall, the results are not fully in line with Zhang and Wang's (2005) findings for internally represented two-digit numbers, which supported the holistic model but also provided some evidence for parallel processing. Zhang and Wang's explanation of digit interference with incomplete internalization of a fixed comparison standard was not applicable to the current study, as there was no fixed standard. Incomplete internalization in general remained possible, because incorrect answers were excluded, but there may have been correct guesses.

Maybe there were design flaws in the study, for example in the method how the stimuli were constructed or in the procedure, whose short-term memory demands may have confounded small differences in response times between number pairs of different decades distances. Maybe participants employed the strategy to memorize the position of the smaller or larger number, instead of memorizing the numbers, to reduce the amount of information processing with increased skill in the task, in line with observations by Haider and Frensch (1996; 1999a and b as cited in Green and Gilhooly, 2005). On the other hand, the experiment detected a unit effect, which is also a small effect and would have been subject to the same strategy.

A decade effect would have been predicted by all models of number processing (holistic, parallel and sequential), therefore evidence for a decade effect would not have helped in ruling out the other models in favour of the holistic processing model. Maybe none of the models is able to fully explain the processing of internally represented two-digit Arabic numbers. A further possibility is that abandoning a fixed standard was responsible for the disappearance of the decade effect. Further research may help to clarify under which circumstances a decade effect does or does not appear. A symmetrical experiment for external representations, presenting the task before the numbers, might show if the decade effect was only absent for internal representations.

References

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